Introduction:

The use of vector sum analysis in determining momentum plays a very important role in accident reconstruction. To understand vectors and their components is to understand momentum, and a graphical solution of momentum with vector analysis provides a picture that will aid in this understanding. In our collision analysis we will use two vehicles only.

Vector analysis not only yields a graphical solution, but also:

1) Gives the reconstructionist a method to check the mathematics of the linear momentum equation.
2) Shows the change in momentum ($\Delta \mathbf{M}$) of each vehicle, which according to Newton's 3rd Law must be equal and opposite to the change in momentum of the other vehicle. The change in velocity ($\Delta \mathbf{v}$) of each vehicle can also be determined.

The change in momentum ($\Delta \mathbf{M}$) is also referred to as the **impulse**, which is the force that will redirect the vehicle. Impulse is based on Newton's 2nd law:

$$\mathbf{F} = m\mathbf{a}$$

Therefore, the formula for **impulse** would be:

$$\mathbf{F}_1 \Delta t = m_1 \Delta \mathbf{v}_1$$

Impulse = Change in momentum

(single vehicle)

Original momentum + Impulse = Final momentum

In a two vehicle problem the change in momentum is the result of the impulse placed on each vehicle by the other vehicle. Since it is known that Newton's 2nd Law for each vehicle is:

$$\mathbf{F}_1 \Delta t = m_1 \Delta \mathbf{v}_1$$
$$\mathbf{F}_2 \Delta t = m_2 \Delta \mathbf{v}_2$$

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And Newton's 3rd Law yields:

$$\mathbf{F}_1 \Delta t = -\mathbf{F}_2 \Delta t$$

Then:

$$-\mathbf{F}_2 \Delta t = m_1 \Delta \mathbf{v}_1$$

Therefore, the following issue would be true:

$$-\mathbf{F}_2 \Delta t + \mathbf{F}_2 \Delta t = m_1 \Delta \mathbf{v}_1 + m_2 \Delta \mathbf{v}_2$$

Left side cancels out; rearrange:

$$m_1 \Delta \mathbf{v}_1 + m_2 \Delta \mathbf{v}_2 = 0$$

$\Delta \mathbf{v}_1$ represents the change in velocity from $\mathbf{v}_1$ to $\mathbf{v}_3$ expressed as $\mathbf{v}_3 - \mathbf{v}_1$.

$\Delta \mathbf{v}_2$ represents the change in velocity from $\mathbf{v}_2$ to $\mathbf{v}_4$ expressed as $\mathbf{v}_4 - \mathbf{v}_2$.

Substitute these values for $\Delta \mathbf{v}_1$ and $\Delta \mathbf{v}_2$:

$$m_1(\mathbf{v}_3 - \mathbf{v}_1) + m_2(\mathbf{v}_4 - \mathbf{v}_2) = 0$$

$$m_1 \mathbf{v}_3 - m_1 \mathbf{v}_1 + m_2 \mathbf{v}_4 - m_2 \mathbf{v}_2 = 0$$

Add $m_1 \mathbf{v}_1$ and $m_2 \mathbf{v}_2$ to both sides:

$$m_1 \mathbf{v}_3 + m_2 \mathbf{v}_4 = m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2$$

Since $m = \frac{w}{g}$, substitute this value for m:

$$\frac{w_1 \mathbf{v}_1}{g} + \frac{w_2 \mathbf{v}_2}{g} = \frac{w_1 \mathbf{v}_3}{g} + \frac{w_2 \mathbf{v}_4}{g}$$

Multiply through by g:

$$w_1 \mathbf{v}_1 + w_2 \mathbf{v}_2 = w_1 \mathbf{v}_3 + w_2 \mathbf{v}_4$$

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The importance of impulse cannot be overlooked. Remember, impulse is what redirects the vehicle.

The foregoing sets up $\Delta \mathbf{v}$ and $\Delta \mathbf{M}$ and the momentum equation, all of which will be used in the method that follows.

Post-Collision Scene Data and Trajectory Analysis Using the Right Hand Coordinate System (as used by automotive engineers):

The rectangular coordinate graph has two mutually perpendicular lines marked off to scale. The two lines intersect at the zero (0) point called the **origin.** In the right hand coordinate system, the vertical scale is the x-axis while the horizontal scale is the y-axis. Every point on the plane is designated with a pair of coordinates (x,y) which give the distances from the two axes. Coordinate x gives the distance a vehicle traveled from the y-axis in units of the x-axis, while coordinate y gives the distance from the x-axis in units of the y-axis. The units will be positive for movement up the x-axis or to the right on the y-axis; conversely the units will be negative for movement down the x-axis or left on the y-axis. For example, (1,2) are the coordinates of a point which has one unit in the positive direction x and two units in the positive direction y. Most computer programs are written for a right-hand coordinate system.

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