

ANATOMY OF THE COLLISION

VARIATIONS ON A THEME

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Summary: Eight collisions, which include forty-four variants, represent an overview of the Conservation of Energy and Momentum and the mechanics of Restitution as applied to vehicular collisions. Each collision will be analyzed using Energy, Momentum and Restitution for similarities and differences regarding velocities, changes in velocity or ΔV and damage.

Inertia: The tendency of an object at rest to remain at rest, and of an object in motion to continue in motion without a change in the vector velocity.

Newton's First Law of Motion: An unaccelerated body remains unaccelerated unless it is caused to change that state by forces exerted on it by the environment.

Newton's Second Law of Motion: The acceleration of a particle is equal to the ratio of the net force acting on the particle to the inertial mass of the particle.

Newton's Third Law of Motion: To every action there is always opposed and equal reaction. The action forces and the reaction forces are on different bodies.

Kinetic Energy Formula: 0.5 times MASS times VELOCITY squared ($Ke = 0.5MV^2$).

Conservation of Momentum: The momentum after collision is equal in magnitude and direction to the momentum before the collision ($\mathbf{Pr}' = \mathbf{Pr}$).

Simplifying Assumptions:

- There is no grade;
- The overlap is 100% (inline);
- Opposite side but otherwise identical areas of the vehicle are in contact (oblique);
- Except for mass, the vehicles are absolutely identical before impact;
- The sum of the external forces, including ground forces acting on the vehicles, is zero.

ANATOMY OF THE COLLISION - Variations on a Theme

A total of 8 collisions with 44 inline and oblique variants will be examined. 22 of the variants will have a restitution value of 0, and 22 will have a restitution value of 1. 21 variants will have vehicles with identical masses and 21 will have vehicles with different masses. The remaining 2 variants will be into fixed immovable non-deforming barriers.

In order to make comparisons of the collisions easier, a numbering system will be employed that uses a collision number, a separating hyphen, the restitution value, a second separating hyphen followed by the letter A, B or C. The letter A will designate collisions between vehicles with identical masses. B ($M_1 = 1, M_2 = 2$) and C ($M_1 = 2, M_2 = 1$) will designate collisions between vehicles with different masses.

When dealing with a restitution value of 1 all of the energy from motion into collision must be returned to the system in the form of motion. If the vehicles have different masses, how is this energy returned to the system? Do the vehicles share this energy equally? Do they share the energy by mass? If they share the energy by mass, does the heavier object get the most energy or does the lighter object get the most energy?

Vehicle 1 has a mass of 1. Vehicle 2 has a mass of 2. The amount of energy from restitution used to accelerate the vehicles apart after maximum engagement is 1200 units. Newton's Third Law states that to every action there is always opposed and equal reaction. The action forces and the reaction forces are on different bodies. Newton's Second Law states that the acceleration of a particle is equal to the ratio of the net force acting on the particle to the inertial mass of the particle. $A = F / M$

- If they share the energy equally, each vehicle accelerates gaining 600 units of energy. Vehicle 1 will accelerate to 34.641 v-units per second. Vehicle 2 will accelerate to 24.4949 v-units per second.
- If the energy is shared by mass with the heavier vehicle gaining the greater energy then the total energy, 1200 units multiplied by $2/3$ results in 800 units of energy gained by accelerating the heavier Vehicle 2. The remaining 400 units are gained by accelerating the lighter Vehicle 1. Vehicle 2 accelerates to 28.2843 v-units per second. Vehicle 1 accelerates to 28.2843 v-units per second.
- If the energy is shared by mass with the lighter vehicle gaining the greater energy then the total energy, 1200 units multiplied by $2/3$ results in 800 units of energy gained by accelerating the lighter Vehicle 1. The remaining 400 units are gained by accelerating the heavier Vehicle 2. Vehicle 1 accelerates to 40 v-units per second. Vehicle 2 accelerates to 20 v-units per second.

The force acting on the vehicles must be equal and opposite according to Newton's Third Law. If the mass of Vehicle 1 is only half of the mass of Vehicle 2 then Vehicle 1 will accelerate to twice of the velocity of Vehicle 2 according to Newton's Second Law. The third example is the only one that satisfies Newton's Second Law. Therefore, the percent of the total energy of restitution gained by accelerating each of the objects is inversely proportional to the mass of the objects.

ANATOMY OF THE COLLISION - Variations on a Theme

RESTITUTION:

In a totally **plastic** collision having a restitution value of 0, none of the energy of damage, heat, light or sound is returned to the system. There is permanent damage to the vehicles but no temporary damage. The vehicles reach a common velocity in the collision and none of the energy of damage is returned to the system.

In a totally **elastic** collision having a restitution value of 1, all of the energy at impact is returned to the system. There is no permanent damage to any of the vehicles. This does not mean that everything returns to the pre-collision status. The vehicles reach a common velocity in the collision and then return all energy (including damage if any) to the system through a change of speed, direction, or both.

Determining Velocity and Direction from Momentum:

The post-impact resultant vector must equal the pre-impact resultant vector ($\mathbf{Pr}' = \mathbf{Pr}$). The resultant vectors are a result of vector addition of the individual momentum vectors ($\mathbf{P1} + \mathbf{P2}$ and $\mathbf{P1}' + \mathbf{P2}'$). The individual momentum **vector** denotes both direction and magnitude. Momentum results from multiplying the mass times the velocity of the object or vehicle. Because the total momentum before impact must equal the total momentum after impact, the post-impact velocities for all of the collisions can be computed since the individual momentum vectors and the masses of the vehicles are known. If momentum is a result of multiplying the mass times velocity ($\mathbf{P} = M * \mathbf{V}$), velocity can be obtained by dividing momentum by the mass ($\mathbf{V} = \mathbf{P} / M$).

Energy from Motion: Kinetic Energy = $0.5 * \text{Mass} * \text{Velocity}^2$

Motion from Energy: Velocity = $\text{SQR}(\text{Kinetic Energy} / (0.5 * \text{Mass}))$

Time's Arrow - which way to go?

Reconstructionists normally study a collision in a direction opposite the time arrow. The post-impact data is used to determine what happened at impact. In analyzing the following collisions a different approach will be utilized. It is the effects of changes in velocity, mass, and angle that are the primary focus of the examination of the physics of the collision. In order to compare and contrast the effect of changes in velocity, mass and angle on the collision it is necessary to control these variables at the front (pre-impact side) of the collision working in the direction of the time arrow. The tools and methods employed for examining these collisions are also applicable when the order of the examination is reversed.