Solving Second-Degree and Systems of Equations: Applications to Motor Vehicle Accident Reconstruction

Andrew S. Rich

Introduction

This article begins with a discussion of the methods that may be used to solve second-degree equations, including the quadratic equation. The information is used as a foundation for the discussion of systems of equations. The knowledge gained is then applied to the conservation of linear momentum and the conservation of energy in order to solve collinear momentum problems where none of the impact velocities are known. These procedures are by no means new; they are perhaps as old as algebra itself and have been used by reconstructionists in the past. It is the intent of this article to give those who have not already been exposed to the topic a firm background in what can be a tedious subject.

Second-Degree Equations

The most common type of second degree equation we encounter as reconstructionists is one that has a variable raised to the second power, such as v in $KE = .5mv^2$. A simple second-degree equation is $x^2 = 9$. We can solve this equation easily by taking the square root of both sides: $\sqrt{x^2} = \pm\sqrt{9}$. Therefore, $x = 3$ AND $\text{x} = \text{-3}$, since both $(-3)^2$ and $3^2 = 9$. This method of solving a second-degree equation is known as the square root method. We could also have solved this equation with the factoring method.

Our original equation:	$x^2 = 9$
Subtract 9 from both sides of the equation	$x^2 - 9 = 0$
Factor the left side of the equation	$(x+3)(x-3) = 0$
Find the numbers that will make the left side of the equation equal 0	$\{-3,3\}$

Just like when taking the square root of both sides, our answer is $x = 3$ AND $\text{x} = \text{-3}$.

While the square root method is easier, consider the equation $x^2 = 2x$. Here, we can't just take the square root of both sides. Nor can we just divide both sides by x to get $x = 2$. While 2 is part of the solution, it is not the complete solution. We proceed as follows:

Our equation:	$x^2 = 2x$
Subtract $2x$ from both sides	$x^2 - 2x = 0$
Factor out x from the left side	$x(x-2) = 0$
Find the numbers that will make the left side of the equation equal 0	$\{0,2\}$

Thus, the solutions to $x^2 = 2x$ are 0 and 2. Check the solutions: substitute 2 for x in the original equation and see if it satisfies the equation. Do the same for 0. Notice that a second-degree equation gives us two solutions. How about $x^2 + 8x + 15 = 0$? The square root method will not work. We can, however, factor this equation into $(x+3)(x+5)$. This is done by finding two numbers whose product is the third term (15), and whose sum is the coefficient of the middle term (8). The two numbers are 3 and 5. Once factored, it is easy to see our solution set is –3 and –5. Check this one too: substitute –3 into the original equation and see if it satisfies the equation. Do the same for –5.

So far, we have looked at the easy ones. How about an equation such as $x^2 + x - 5 = 0$? We cannot take the square root of both sides, nor can we factor. We can, however, use the quadratic equation: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. To use the quadratic equation, we must write our second-degree equation in standard form. That is, from the second power to the zero power. The standard form is written $Ax^2 + Bx^1 + Cx^0 = 0$. Since $x^1 = x$, and $x^0 = 1$, we can simplify this to $Ax^2 + Bx + C = 0$. Let's use our previous example, $x^2 + x - 5 = 0$, to demonstrate the quadratic equation. First, we see that the equation is already in standard form. Therefore, our A, B and C coefficients are 1, 1 and –5, respectively. Substitute these numbers into the quadratic equation:

Substitute the coefficients into the quadratic equation	$x = \frac{-1 \pm \sqrt{1^2 - (4)(1)(-5)}}{(2)(1)}$
Complete the exponent and multiplications	$x = \frac{-1 \pm \sqrt{1+20}}{2}$
This is the answer (it is acceptable to leave the answer in this form because $\sqrt{21}$ is an irrational number, which can only be approximated). Completing the arithmetic would give the solution set of {1.79, -2.79}.	$x = \frac{-1 \pm \sqrt{21}}{2}$

Although factoring and the square root method are the easiest to use, they do not work for all second-degree equations. The quadratic equation will work in all cases, so it is essential to know.

Systems of Equations

As you progress through the paragraph below, it will become clear what a system of equations is. Let's look at a situation where a system of equations may be applied. You are asked to calculate the velocity of two cars, given the following information:

- Both cars are traveling directly towards each other, at the same time, from positions 480 miles apart
- They meet after 4 hours
- Vehicle 1 is traveling 40 mph faster than vehicle 2

One equation will not solve this problem for us. The most obvious equation that we can apply here comes from information that vehicle 1 is traveling 40 mph faster than vehicle 2. To show this algebraically, we write the equation $v_1 = v_2 + 40$ **(Equation A)**. We can see that there are an infinite number of solutions to this equation: 80=40+40, 70=30+40, 100=60+40, etc. We obviously need more information to solve this problem; this information is provided by the first two clues. Recall that distance = velocity X time, or $d = vt$. When we picture in our minds the two cars, we know that they will meet after 4 hours. We also know that because vehicle 1 is traveling faster than vehicle 2, it will cover more distance during this time. We can say with certainty that the distance vehicle 1 travels plus the distance vehicle 2 travels will equal 480 miles. Written algebraically, $480 = d_1 + d_2$. We know that $d = vt$, so let's substitute vt for d: $480 = v_1 t + v_2 t$. Since they both start at the same time, and meet at the same time, the time for both vehicles is the same in our equation. Before proceeding further, factor the t in the right side of the equation: $480 = t(v_1 + v_2)$ **(Equation B)**. Even this equation, by itself, does not help us since there are infinite solutions. Let's call on Equation A to give us a hand. We will substitute equation A – solved for v_1 – into Equation B. When we do this, we have an equation that looks like this: $480 = t(v_2 + 40 + v_2)$. The time in this case is 4 hours; therefore, we substitute 4 for t giving $480 = 4(v_2 + 40 + v_2)$. Now we have an equation we can solve because it only has one variable.

Our original equation:	$480 = 4(v_2 + 40 + v_2)$
Combine like-terms	$480 = 4(2v_2 + 40)$
Distribute the 4 to the right side	$480 = 8v_2 + 160$
Subtract 160 from both sides	$320 = 8v_2$
Divide both sides by 8	$v_2 = 40$