Chapter 1

Mathematical Concepts

Many people who get into traffic crash reconstruction may not have a strong mathematical background or have been out of school for many years. Therefore, this chapter aims at bringing the reader up to speed with some of the concepts of mathematics used in the field of reconstruction. Some preliminary topics are covered quickly as a review; however, the later sections focus on expanding the reader's understanding of applied mathematics.

1.1 Arithmetic

1.1.1 Working with Signed Numbers

When we work with numbers, not all the numbers are going to be positive. When working with signed numbers, we need to follow certain rules to arrive at the correct answer.

Adding

- When adding like signs, add the numbers and carry the sign.
- When adding unlike signs, take the difference and give the answer the sign of the dominant (larger) number.

Subtracting

Change the sign of the subtrahend (the number being subtracted) and follow the rules of addition.

Multiplying

Show the product of the numbers (the result of the multiplication) and

- if the numbers have like signs, the answer will always be positive.
- if the numbers have unlike signs, the answer will always be negative.

Dividing

Show the quotient of the numbers (the result of the division) and follow the rules of multiplication.

Example 1.1 *Working with signed numbers.*

$$3 + 5 = 8 \qquad -6 - 3 = -9$$
$$3 + (-8) = -5 \qquad 7 \times 2 = 14$$

Parentheses

When a negative sign precedes an expression in parentheses, then every term in the expression is negated, which means the sign on every term is switched. A term can be either a single value, a product, or a quotient. Another way to think of it is multiplying every term in the parentheses by -1.

Example 1.2 *Negative parentheses.*

$$-(1 + 2) = -1 - 2 = -3$$
$$-(3 - 4) = -3 + 4 = 1$$
$$-\left(\frac{1}{2} - 2 \times 3\right) = -\frac{1}{2} + 2 \times 3 = 5.5$$

Exponents

Exponents are a short way of writing a series of multiplications of the same number.

$$3 \times 3 \times 3 = 3^3 = 27$$

In this sense, we just follow the rules of multiplication. However, those rules can be simplified based on whether the exponent is odd or even. If the exponent of a negative number is:

- odd, then the result of the exponentiation will be negative.
- even, then the result of the exponentiation will be positive.

Raising a negative number to an exponent is only permitted if the exponent is an integer. An *integer* is any number without a decimal point, $(\cdots, -3, -2, -1, 0, 1, 2, 3, \cdots)$. The three center dots mean to continue forever in the same pattern. If the exponent of a negative number is not an integer, the solution is either complex or imaginary, which never happens in traffic crash reconstruction.

Example 1.3 *Negative numbers raised to a power.*

$$(-5)^2 = -5 \times -5 = 25$$
$$(-10)^3 = -10 \times -10 \times -10 = -1000$$

These rules lead to the observation:

The square of any number is non-negative.

Sometimes negative numbers appear in the exponent. Raising any number to a negative exponent is the same as dividing the result of the positive power into 1.

$$a^{-b} = \frac{1}{a^b} \tag{1.1}$$

Example 1.4 *Dealing with a negative power.*

$$2^{-3} = \frac{1}{2^3} = \frac{1}{8} = 0.125$$
$$10^{-4} = \frac{1}{10^{-4}} = \frac{1}{10,000} = 0.0001$$

1.1.2 Order of Operations

An order of operation exists that defines which arithmetic operation takes precedence over another when different operators occur in a single expression. This order can be recalled by a little memory jogger: